

Teacher as a designer of learning experiences: Using GeoGebra to support visualisation in geometry teaching

Melisa Husejnović

JU Druga osnovna škola Brčko, Bosnia and Herzegovina

*Correspondence: alicmelisa7@gmail.com

Abstract: This paper presents the use of GeoGebra as an innovative digital tool for improving the understanding of geometric concepts in primary education. Relying on Van Hiele's theory of geometric thinking, the paper illustrates how GeoGebra supports the development of students' geometric reasoning through dynamic visualisation and exploration activities. Selected classroom examples demonstrate that GeoGebra facilitates clearer visual representations of abstract concepts, supports informal deductive reasoning, and reduces the technical demands on teachers. The paper concludes that GeoGebra represents an effective and practical innovation in geometry teaching, benefiting both students' learning and teachers' instructional practice.

Keywords: GeoGebra; Van Hiele theory; Geometric thinking, Visualisation

1. Introduction

Geometry first emerged as a scientific discipline in the ancient civilizations of Egypt, Babylon, and Ancient Greece, closely connected to the need for measuring land, space, and objects in everyday life. The very term *geometry* derives from the Greek words *geo* (earth) and *metron* (measure), clearly indicating its practical origins. It was in Ancient Greece that geometry acquired an entirely new dimension, developing into an abstract and rigorously logical branch of mathematics based on proofs, definitions and theorems.

Throughout history, geometry has been regarded as one of the most demanding areas of mathematics. Its genuine understanding has been associated with higher levels of thinking, abstraction and logical reasoning. It is interesting to consider how today, upon merely hearing terms such as prism, pyramid or cone, a clear spatial image almost instantly forms in our minds. This ability to visualise does not exist at birth, nor does it develop spontaneously; rather, it is the result of a long process of learning, observation and cognitive development. Consequently, the way in which students gradually acquire geometric concepts represents one of the key challenges in mathematics education.

Observing that students often experience significant difficulties in understanding geometry—not due to a lack of knowledge, but rather due to the level of thinking they have

reached—Dina and Pierre van Hiele developed a theory of cognitive development in geometry [1,2]. Van Hiele's theory explains how students progress through clearly defined levels of geometric thinking. According to this theory, students cannot understand geometric proofs unless they have previously developed the appropriate level of reasoning, regardless of their age or the amount of instruction they have received.

In contemporary mathematics education, particularly in the context of digitalisation, there is a growing need for tools that support this gradual development of geometric thinking. Within this context, GeoGebra emerges as a highly effective didactic tool [3].

Connecting Van Hiele's theoretical framework with the use of GeoGebra enables teachers to adapt geometry instruction to students' cognitive abilities and to guide them progressively from visual recognition toward deeper understanding and formal reasoning. Such an approach not only facilitates the learning of geometry, but also transforms the role of students—from passive recipients of information into active explorers of mathematical relationships.

The aim of this paper is to illustrate how GeoGebra can support the development of geometric thinking in accordance with Van Hiele's theory through selected classroom examples.

In addition, the paper discusses the pedagogical value of dynamic visualization as a means of supporting inquiry-based learning and facilitating the gradual transition from intuitive geometric perception to conceptual understanding.

2. Van Hiele's theoretical framework

Van Hiele's theory of learning geometry explains why many students experience difficulties in understanding geometric concepts, particularly in performing formal proofs. According to this theory, students progress through a sequence of five successive levels of geometric thinking:

1. **Visualisation:** Students recognise geometric figures based on their appearance, without understanding their properties;
2. **Analysis:** Students describe figures by listing their properties, but do not yet understand the relationships between them or distinguish necessary from sufficient conditions;
3. **Informal Deduction:** Students connect properties and recognise relationships between figures, identifying sufficient conditions for defining a geometric shape;

4. **Formal Deduction:** Students understand the role of proof and are capable of independently proving mathematical statements;
5. **Rigor:** Students analyse axiomatic systems and compare Euclidean and non-Euclidean geometries as coherent mathematical structures [1,2].

The following example illustrates these levels using a concrete geometric figure.

Table 1. Van Hiele levels of geometric thinking: The case of the parallelogram

Level 1	Level 2	Level 3	Level 4	Level 5
This is a parallelogram because it looks like a slanted rectangle.	The student knows that a parallelogram has two pairs of parallel sides, that the opposite sides are equal, and that the diagonals bisect each other.	The student concludes: "If a quadrilateral has two pairs of parallel sides, then it must be a parallelogram," or "a rectangle is a special case of a parallelogram."	The student is able to prove a theorem, for example: "If a quadrilateral has two pairs of parallel sides, then its opposite angles are equal, and its diagonals bisect each other," using definitions, axioms, and previously proven statements.	The student understands that the properties of a parallelogram can differ across different axiomatic systems, compares Euclidean and non-Euclidean geometry, and reflects on which axioms are used and why.
The shape is recognized by its appearance, but if you rotate it or change its proportions, students are often no longer sure what they are looking at.	The student does not yet see the connection between these properties—they are "listed," but not logically linked.	Here, an understanding of the relationships between figures and their properties emerges, but without formal proof.	At this stage, geometry becomes a system rather than a collection of drawings.	Here, it is not only the statement that is being proved—the system of proof itself is being analyzed.

Within the framework of Van Hiele's theory of geometric thinking, the goal of geometry teaching in primary education is not the attainment of formal proof, but the gradual development of students' understanding of geometric concepts and their mutual relationships. It is realistic to expect primary school students to reach the third level of

geometric thinking, at which they are able to recognise, describe, and connect the properties of geometric figures, as well as to understand hierarchical relationships between them, even though they are not yet ready for formal proof. Insisting on higher levels without adequate cognitive preparation may lead to mechanical learning without genuine understanding. Therefore, the role of the teacher is to plan geometry instruction in accordance with students' developmental capacities, using methods that encourage exploration, visualisation and meaningful learning, thereby establishing a solid foundation for later formal geometric reasoning.

3. GeoGebra in Mathematics Teaching

Given the undeniable impact that the rapid development of technology has had on the learning and teaching of mathematical concepts, it is natural that technology has become an integral part of contemporary education. As a dynamic mathematics software, GeoGebra enables students to engage with geometric objects not as static figures, but as entities that can be explored, modified, transformed, and linked to algebraic representations. In this way, abstract concepts become visible, and the learning process becomes exploratory and meaningful [3,6,7]. Its particular strength lies in the simplicity of creating interactive teaching materials, as well as in the possibility of sharing and exchanging them within the teaching community.

Nevertheless, GeoGebra should not be viewed as a substitute for sound pedagogical practice. Its educational value depends primarily on the way learning activities are designed and integrated into classroom instruction. Well-planned tasks, appropriate teacher guidance, and meaningful discussion remain essential for transforming visual exploration into conceptual understanding.

3.1. The Use of GeoGebra in Primary School Mathematics Teaching

The use of GeoGebra in primary school allows the integration of a modern, inquiry-based approach across nearly all geometric topics. Through dynamic constructions, students can explore isometric transformations in the plane, relationships between angles and sides, constructions of geometric figures, as well as spatial relationships and properties of geometric solids. GeoGebra enables geometric objects to be perceived as variable and interconnected, thereby fostering active learning, visual thinking, and a deeper understanding of abstract concepts. The following examples illustrate only a small portion of the possibilities that such an interactive mathematical approach offers in geometry instruction.

3.1.1. The Sum of Interior Angles of a Triangle or a Quadrilateral

As an example of a classroom activity in primary education, an inquiry-based task can be implemented. Students first construct an equilateral triangle (using the concept of a regular polygon) and a scalene triangle in which they vary the lengths of the sides, and then measure the sizes of the interior angles. Afterward, they sum the obtained values and compare the results.

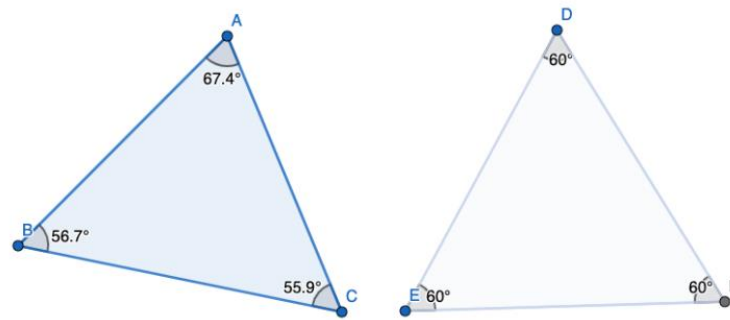


Figure 1. Visual demonstration of the sum of the interior angles of a triangle

Based on repeated measurements and comparisons, students independently formulate the conclusion that the sum of the interior angles of any triangle is equal to 180° . In addition, through this activity, students reinforce their understanding of the relationships between the angles and sides of a triangle and develop a deeper insight into the structure of geometric figures [4,5,7].

The proof that the sum of the interior angles of a quadrilateral is equal to 360° can be presented effectively and visually through the use of pre-prepared dynamic representations.

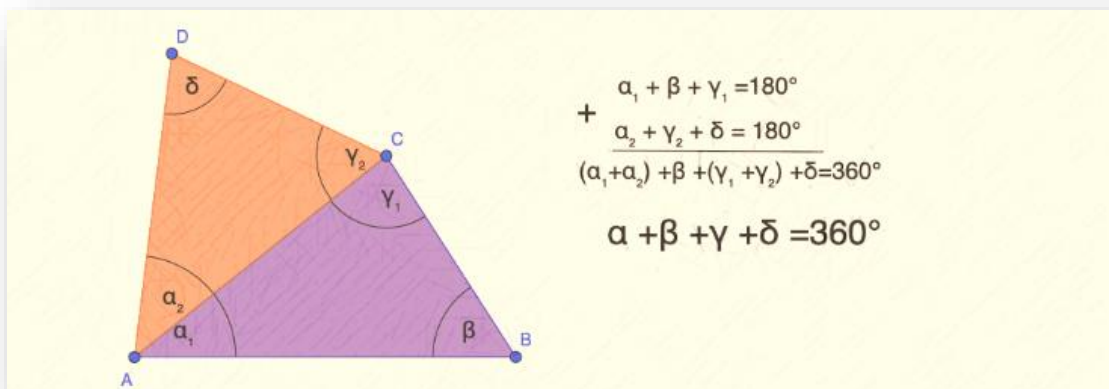


Figure 2. Proof of the sum of the interior angles of a quadrilateral (author: Ed) [7]

3.1.2. The Pythagorean Theorem

Another example of an inquiry-based activity involves investigating the relationships between the sides of a right triangle. After constructing a right triangle and squares on its sides, students are tasked with calculating the areas of the squares and observing how these areas change as the side lengths vary. Through this process, students conclude that the area of the square constructed on the hypotenuse is equal to the sum of the areas of the squares constructed on the legs [3,4,7].

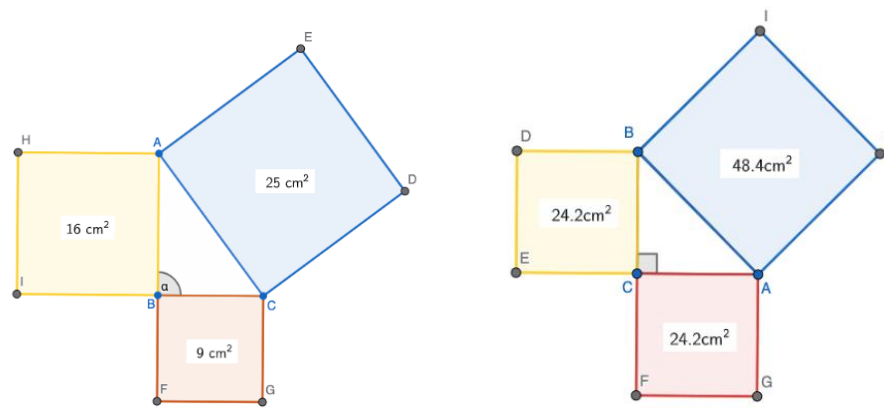


Figure 3. Areas of the squares constructed on the sides of a right triangle

As a geometric proof of the Pythagorean theorem, various pre-prepared animations can be used, enabling a clearer and more time-efficient presentation of the proof. In this way, students can examine multiple different proofs within a short period of time, without lengthy constructions and drawings on the board, allowing instructional focus to shift toward understanding the underlying idea of the proof.

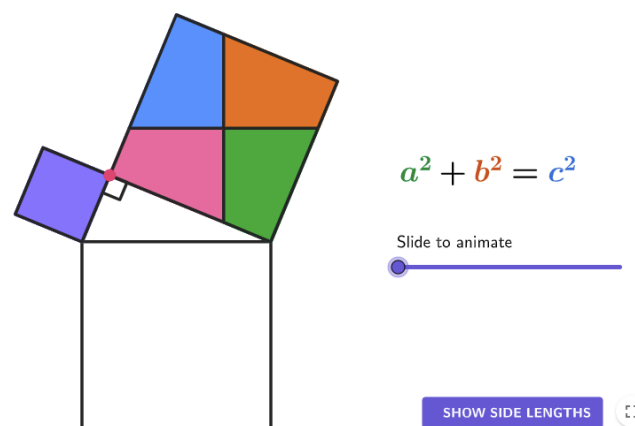


Figure 4. Geometric proof of the Pythagorean theorem (author: GeoGebra content team) [7]

3.1.3. Circumference of a Circle and the Number π

In GeoGebra, it is possible to conduct a measurement-based experiment in which students investigate the relationship between the circumference of a circle and its diameter. By changing the size of the circle, it can be observed that the circumference increases proportionally, while the ratio of the circumference to the diameter remains nearly constant. Students may also interpret this relationship intuitively by considering how many times the length of the diameter “fits” into the circumference. Each measurement yields a value close to 3, with a small remainder, which naturally raises questions about the precision of this relationship. This very impossibility of expressing the ratio of the circumference to the diameter as a finite rational number posed a challenge to mathematicians throughout history, leading to the use of rational approximations, such as the fraction $\frac{22}{7}$, as practical estimates of the value of π . In this way, the number π is introduced to students as a mathematical constant arising from an observed regularity, prior to its formal definition [4,5,7].

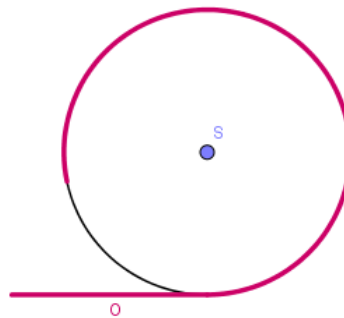


Figure 5. Measuring the circumference of a circle

These examples illustrate that GeoGebra is particularly effective when students are encouraged to formulate hypotheses, test conjectures, and discuss their observations collaboratively. In this way, the software supports not only visualization but also the development of mathematical reasoning and communication skills.

4. Conclusion

Naturally, there are many additional examples of how GeoGebra can be applied in the mathematics classroom, through which this tool proves to be functional and relatively easy to use. The examples presented in this paper demonstrate that the integration of GeoGebra with Van Hiele's theoretical framework provides teachers with a practical methodology for

designing learning experiences that are better aligned with students' cognitive development. Rather than simply presenting geometric facts, such an approach encourages active exploration, conceptual understanding, and gradual progression toward higher levels of geometric reasoning. At the same time, it is important to emphasise that effective implementation requires a certain amount of time for both teachers and students to become familiar with the working environment, and that the success of its use largely depends on the users' level of digital literacy.

Although all of the described activities can also be carried out in a traditional classroom setting, such an approach requires considerably more time and often results in fewer repetitions, which may limit students' ability to recognise general patterns. In such cases, students' attention is frequently shifted from mathematical reasoning to the technical accuracy of drawing and measuring. When classrooms do not have a sufficient number of computers, activities can be successfully adapted through pair or small-group work, with multiple students sharing a single device. Alternatively, the teacher may use GeoGebra via a projector or interactive whiteboard, while students guide the course of the investigation, propose changes to the figures, and jointly draw conclusions. This approach preserves the inquiry-based nature of the activities, encourages active student participation, and ensures the achievement of instructional goals even under technically constrained conditions.

Future research may investigate the long-term impact of GeoGebra-supported instruction on students' geometric achievement, motivation, and problem-solving abilities, as well as the professional development needs of teachers implementing digital technologies in mathematics classrooms.

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